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# THE THEORY OF EFFECTIVE PROTECTION AND RESOURCE ALLOCATION: THE MANY-INDUSTRY CASE*

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Number 182 July 1976

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Abstract

This paper demonstrates that, for the non-separable case, Sendo's sufficiency restriction on change in tariff structure, and therefore also the Bhagwati-Srinivasan sufficiency restriction on tariff change which it extends, is generalizable from the two-industry case to the many-industry case. ERP theory, in the sense defined by Bhagwati and Srinivasan, is thus shown to hold, for the Sendo tariff structure, within the framework of the Bhagwati-Srinivasan general equilibrium model with many industries, many primary factors, many imported inputs and non-separable production functions.

## 1. Introduction

The theory of effective protection (ERP) has been developed in an attempt to seek a concept of protection which, in the presence of imports of intermediates, will perform the same role in predicting the effects of a change in tariff structure on domestic resource allocation as nominal tariffs do in their absence. The problem of generalization of the theory from the assumption of fixed production coefficients in respect of intermediates towards allowing their substitution in production in a general equilibrium model has received the attention of several economists.[1] The studies of Bruno (1973) and Bhagwati and Srinivasan (1973) on the effect of tariff change in a general equilibrium model with n industries, d primary factors and m imported inputs are particularly significant in this connection.

Bruno defines "real" value added as the difference between the value at free trade prices of post-tariff outputs and non-factor inputs, that is, $F = \{X - e'M\}$ where X is output, M a column vector of non-factor tradeable inputs in the post-tariff situation, and e' the row vector (1, ...,1) where the world prices are normalized at unity. On the other hand, nominal value added is $G = \{(1+T)X - (e+t)'M\}$ where T is the ad valorem tariff on output and t' the row vector $(t_1,\ldots,t_m)$ of ad valorem tariff on tradeable inputs. He defines the effective rate of protection (EPR) as $B = \{(G/F) - 1\}$. However, Bruno deals mainly with the

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[1] See Bhagwati and Srinivasan (1973), Bruno (1973), Corden (1969, 1971), Jones (1971), Khang (1973), Ramaswami and Srinivasan (1971), Rav (1973) and Sendo (1974).

issue of the conditions under which a tariff structure will increase (decrease) the use of all primary factors in an industry under a general equilibirum framework in which the factor endowment in the economy is exogenously fixed; the ERP defined in the above is discussed only in the case of functional separability.

The B-S analysis, on the other hand, is directly focused on ERP theory. They observe that the proportionate change of nominal value added $V_i$ $(i=1,\ldots,n)$ (Bruno's G) induced by a change in tariff structure can be expressed as the sum of two terms: a Divisia index (I) of proportionate changes in all the prices of output and imported inputs faced by the ith industry, and a Divisia index (II) of proportionate changes in the quantities of the factors used by the ith industry. Then they define the ERP problem as one of investigating whether the ERP (Divisia) index (I) of price changes and the Divisia index (II) of quantity changes will move in the same direction for an activity. They propose two approaches to this problem.

The first approach is to look for restrictions on production functions that will be sufficient for indexes I and II to move in the same direction. This led Bhagwati and Srinivasan to the condition of functional separability.

The second approach, on the other hand, is to look for restrictions on tariff changes that will equally suffice to make indexes I and II move in the same direction. Since no special restrictions on production functions can be used in this alternative approach, one can expect that the restrictions required on tariff changes would have to be rather strong. Essentially, Bhagwati and Srinivasan looked for

tariff changes which would result in a positive flow of at least one domestic resource (and no withdrawal of any other resource) to the activity whose (price) index (I) was relatively higher than in other activities thanks to the tariff change, thus sufficing to make the (quantity) index (II) positive as well. This led them to the following restriction on tariff changes:

$$\text{(B-S)} \qquad \hat{P}_1^{\circ} >(<) \hat{P}_i^{\circ} = \hat{P}_k^{M}, \; i = 2,\ldots,n; k = 1,\ldots,m$$

where $P_i^{\circ}$ and $P_k^{M}$ denote the domestic prices of the ith good and the kth imported input respectively, and $\hat{z} \equiv dz/z$ for any variable z. Furthermore, this sufficiency condition was established only for the two-industry case. For the many-industry case (n>2), they managed to show that condition (B-S) above is sufficient for index (I) and the change in <u>gross outputs</u> of the first activity to have the same sign, but they were unable to prove anything in general about the sign of index (II).

The B-S sufficiency restrictions on tariff changes were then generalized by Sendo (1974) for the two-industry case as follows:

$$\text{(S)} \qquad \hat{P}_1^{\circ} \geqq P_k^{M} \geqq \hat{P}_2^{\circ} \qquad k = 1,\ldots,m$$

with at least one strict inequality holding. In this case, semi-positive resource flows to industry 1 will take place and index (II) will be positive (along with index (I), absolutely and relatively to industry 2) in industry 1. This condition was anticipated by Bruno (1973) for the two-industry case,(though in Bruno's model, the tariffs on imported intermediates can vary from industry to industry whereas they are identical in all industries in the Bhagwati-Srinivasan model).

Bruno (1973) also attempted to investigate the many-industry case, assuming the case where one industry's output and/or input tariffs have been changed leaving all tariffs in the rest of the economy unchanged. Then by placing the following additional restrictions on production functions, he concluded that all factors move to or away from the first industry. These conditions are:

(i) The production functions are strictly decreasing returns to scale.

(ii) $H \equiv (A^2)^{-1} + \ldots + (A^n)^{-1}$ has the Metzler property, that is, the diagonal elements of $H^{-1}$ are negative and the off-diagonal ones are nonnegative, where $A^i$ is the Hessian of the ith nominal value added function G with respect to its primary inputs.

Bruno's theorem however is not appealing. Firstly, it will not hold if the production functions are constant returns to scale. Secondly, condition (ii) is in terms of the sum of (n-1) Hessians of value-added functions (except for the first sector) and therefore, its economic interpretation is by no means clear.

In this paper, we revert to the original Bhagwati-Srinivasan approach and manage to show that, for the tariff-structure change (S) stated by Sendo and encompassing the (B-S) sufficiency condition, ERP theory will work in the sense defined by Bhagwati and Srinivasan, not merely for the two-industry but also for the many-industry case. Thus if protection is conferred on, say, the first industry by the tariff structure, so that its Divisia index (I), is (absolutely and relatively) positive, then there will be a semi-positive resource flow into this industry and out of the rest of the economy, and its Divisia

index (II) will then be positive. It follows then that this protected first industry's nominal value added will also rise; we also show, as Bhagwati and Srinivasan earlier did, for the many-industry case, that the first industry's gross output will also rise.

## 2. The Model

B-S model (1973) is the following: Let the production function for the ith good be $F^i\ (D^i, M^i)$ where $D^i = (D^i_1, \ldots, D^i_d)'$ is the column vector of domestic inputs and $M^i = (M^i_1, \ldots, M^i_m)'$ is the column vector of imported inputs.

It is assumed that:

(A-1) All inputs enter into the production of each commodity.

(A-2) Each production function is homogeneous of degree one, concave and its Hessian matrix is indecomposable with negative diagonal and nonnegative off-diagonal elements.[2]

(A-3) Production takes place under perfect competition, given the domestic prices vector, $P^\circ = (P^\circ_1, \ldots, P^\circ_n)'$ and $P^M = (P^M_1, \ldots, P^M_m)'$, respectively of the outputs and imported inputs.

(A-4) Every commodity is produced.

(A-5) The rank of the primary input coefficient matrix, $[D^1, \ldots D^n]$ is n.

In this section we need the following notation due to B-S.

(1) $F^i_D = (\partial F^i/\partial D^i_1, \ldots, \partial F^i/\partial D^i_d)'$, $\quad i = 1, \ldots, n$

(2) $F^i_M = (\partial F^i/\partial M^i_1, \ldots, \partial F^i/\partial M^i_m)'$, $\quad i = 1, \ldots, n$

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[2] B-S assume that the Hessian of each production function has negative diagonal and positive off-diagonal elements.

(3) $F^i_{DD} = (\partial^2 F^i/\partial D^i_j \partial D^i_k)$, $\quad i = 1,\ldots,n;\ j,k = 1,\ldots,d$

(4) $F^i_{DM} = (\partial^2 F^i/\partial M^i_k \partial D^i_j)$, $\quad i = 1,\ldots,n;\ j = 1,\ldots,d;\ k = 1,\ldots,m$

(5) $F^i_{MD} = (F^i_{DM})'$,

(6) $F^i_{MM} = (\partial^2 F^i/\partial M^i_j \partial M^i_k)$, $\quad i = 1,\ldots,n;\ j,k = 1,\ldots,m$

The competitive equilibrium conditions are:

(7) $P^o_i F^i_D = W \qquad i = 1,\ldots,n$

(8) $P^o_i F^i_M = P^M, \qquad i = 1,\ldots,n$

(9) $\sum_{i=1}^{n} D^i = \bar{D}$

where $\bar{D} = (\bar{D}_1,\ldots,\bar{D}_d)$ is the endowment vector of the primary factors in the economy and W a primary factor price vector.

Differentiating equations (7)-(9) totally, we get

(10) $P^o_i[F^i_{DD}\dot{D}^i + F^i_{DM}\dot{M}^i] + \hat{P}^o_i W = \dot{W} \qquad i = 1,\ldots,n$

(11) $P^o_i[F^i_{MD}\dot{D}^i + F^i_{MM}\dot{M}^i] + \hat{P}^o_i P^M = [\bar{P}^M]\hat{P}^M \qquad i = 1,\ldots,n$

(12) $\sum_{i=1}^{n} \dot{D}^i = 0$

where, for any z, $\dot{z} \equiv dz$ and $\hat{z} \equiv \dot{z}/z$, and $[\bar{P}^M]$ is the diagonal matrix whose (i,i)th element is $P^M_i$ (i=1,...,m).

Eliminating $\dot{M}^i$ (i=1,...,n), we get

(13) $A^i\dot{D}^i = -F^i_{DM}(F^i_{MM})^{-1}[\bar{P}^M](\hat{P}^M - \hat{P}^o_i e) - \hat{P}^o_i W + \dot{W}, \quad i = 1,\ldots,n$

where $A^i = P^o_i[F^i_{DD} - F^i_{DM}(F^i_{MM})^{-1}F^i_{MD}]$ and $F^i_{MM}$ is negative definite by (A-2) (see Theorem 1 of Appendix) and therefore, $(F^i_{MM})^{-1} < 0$. Subtracting the ith equation from the first equation in (13), we get

(14) $A^1\dot{D}^1 = A^i\dot{D}^i - b^i, \qquad i = 2,\ldots,n$

where

$$(15)\quad b^i = -F^1_{DM}(F^1_{MM})^{-1}[\bar{P}^M](\hat{P}^\circ_1 e - \hat{P}^M) - F^i_{DM}(F^i_{MM})^{-1}[\bar{P}^M](\hat{P}^M - \hat{P}^\circ_i e)$$

$$+ (\hat{P}^\circ_1 - \hat{P}^\circ_i)W, \qquad i = 2,\ldots,n$$

## 3. Tariff Change and Resource Allocation

Suppose that protection is now conferred only on the first industry by the following change in the tariff structure:

$$(U)\quad \hat{P}^\circ_1 \geq \hat{P}^M_k \geq \hat{P}^\circ_i, \qquad (K=1,\ldots,m;\ i=2,\ldots,n)$$

with at least one strict inequality.

Then we see that[3]

$$(16)\quad b^i > 0, \qquad i = 2,\ldots,n$$

Substituting $\dot{D}^1 = -\sum_{i=2}^{n} \dot{D}^i$ in (14), we get

$$(17)\quad \Delta((\dot{D}^2)',\ldots,(\dot{D}^n)')' = (b^2)',\ldots,(b^n)')'$$

where

$$\Delta \equiv \begin{bmatrix} \Delta_{22} & \cdots & \Delta_{2n} \\ \vdots & & \vdots \\ \Delta_{n2} & \cdots & \Delta_{nn} \end{bmatrix}$$

and the square matrix $\Delta_{ij}$ of order d for $i,j = 2,\ldots,n$ is given by

$$\Delta_{ij} = A^i + A^1 \quad \text{if } i = j$$
$$= A^1 \quad \text{if } i \neq j$$

By condition (A-5), $\Delta$ is negative definite (see Theorem 2 of Appendix). Therefore, equation (17) has a unique non-zero solution $((\dot{D}^2)',\ldots,(\dot{D}^n)')$.

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3 By $x > 0$ we mean: $x_i > 0$ for all i. By $x \geq 0$ we mean; $x_i \geq 0$ for all i and $x_i > 0$ for at least one i. By $x \geqq 0$ we mean: $x_i \geq 0$ for all i.

Hence, $\dot{D}^1$ $(= -\sum_{i=2}^{n} \dot{D}^i)$ is also uniquely determined.

Now, we shall prove that $\dot{D}^1$ is semi-positive. Since $(D^i)'A^i = 0$ $(i=1,\ldots,n)$, we see from (14) that the equation

$$(18) \quad H'A^1\dot{D}^1 = -c$$

has a solution $\dot{D}^1$, where $H = [D^2,\ldots,D^n] > 0$ and $c = ((D^2)'b^2,\ldots,(D^n)'b^n) > 0$.

Suppose equation (18) does not have a nonnegative solution $\dot{D}^1$. Then from the duality,[4] the inequalities

$$(19) \quad y'H'A^1 \geqq 0 \text{ and } y'c > 0$$

have a solution v. Let $z' \equiv y'H'$. Then, since $A^1$ satisfies the hypothesis of Theorem 1 in the Appendix, there exists a unique $z > 0$ such that $z'A^1 = 0$ and $z'e = 1$. But we already know that $(D^1)'A^1 = 0$ and $D^1 > 0$. Hence, <u>but</u> for the normalization $z'e = 1$, we have $z = D^1$. Thus there exists a y such that $z' = y'H'$ and $z'A^1 = 0$. But we have from (18) and (19) that $0 = z'A^1\dot{D}^1 = y'H'A^1\dot{D}^1 = -v'c < 0$, a contradiction. Hence, inequalities (19) have no solution. By duality, equation (18) has a nonnegative solution $\dot{D}^1$. Since $c > 0$, we see from (18) that $\dot{D}^1$ should be semi-positive.

Thus, if protection is conferred only on the first industry by the change in the tariff structure, then there will be a semi-positive

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[4] Exactly one of the following alternatives holds. Either the equation

$$(1) \quad xA = b$$

has a nonnegative solution or the inequalities

$$(2) \quad Av \geqq 0 \text{ and } bv < 0$$

have a solution. (See Gale (1960, Theorem 2.6, page 44).)

resource inflow into this industry and out of the rest of the economy.

The reader should note that the restriction on tariff structure is used only to ensure that $b^i > 0$ (and hence $c > 0$). It is clear from equation (15) that the second term is zero under the (B-S) restriction on tariff change, thus ensuring $b^i > 0$. Under the (S) restriction, as also the many-industry (U) restriction, this second term would be positive, thus only reinforcing the Bhagwati-Srinivasan argument in making $b^i > 0$. [Intuitively also, this relaxation of the (B-S) restriction by the (S) and (U) restrictions on tariff change, is thoroughly plausible. For, while (B-S) restriction confers positive protection to industry 1, holding the protection on industry 2 at zero, the (S) restriction permits industry 2 to have negative protection while the (U) restriction permits one or more of industries 2,...,n to have negative protection. Clearly, therefore, the "pull-effect" of protection to industry 1 can only be reinforced by permitting negative protection to industries other than 1!] It is equally clear that $b^i > 0$ is positive even if the second term is negative but to ensure this in general, one would obviously have to bring in special restrictions on production functions.

## 4. Change in Gross Output and in Nominal Value Added

Let us evaluate the sign of the change in gross output of the first industry, as well as the change in value added by it.

$$(20)\quad \dot{F}^1 = (F_D^1)'\dot{D}^1 + (F_M^1)'\dot{M}^1$$
$$= (F_D^1)'\dot{D}^1 + (F_M^1)'[(1/P_1^\circ)(F_{MM}^1)^{-1}[\bar{P}^M](\hat{P}^M - \hat{P}_1^\circ e) - (F_{MM}^1)^{-1}F_{MD}^1\dot{D}^1]$$
$$= [(F_D^1)' - (F_M^1)'(F_{MM}^1)^{-1}F_{MD}^1]\dot{D}^1 + (1/P_1^\circ)(F_M^1)'(F_{MM}^1)^{-1}[\bar{P}^M](\hat{P}^M - \hat{P}_1^\circ e)$$

Since nominal value added in industry 1 is expressed as $V^1 = P_1^\circ F^1 - (P^M)'M^1$, we get

$$(21)\quad \dot{V}^1 = \hat{P}_1^\circ P_1^\circ[(F_D^1)'D^1 + (F_M^1)'M^1] + P_1^\circ[(F_D^1)'\dot{D}^1 + (F_M^1)'\dot{M}^1]$$
$$-(\hat{P}^M)'[P^M]M^1 - (P^M)'\dot{M}^1$$
$$= \hat{P}_1^\circ W'D^1 + [\hat{P}_1^\circ e' - (\hat{P}^M)'][\bar{P}^M]M^1 + W'\dot{D}^1$$

Furthermore, according to Bhagwati and Srinivasan, the two Divisia indices, $\hat{P}_v^i$ and $\hat{Q}_v^i$ are defined as

$$(22)\quad \hat{V}^i = \hat{P}_v^i + \hat{Q}_v^i$$

where

$$(23)\quad \hat{P}_v^i = \frac{\hat{P}_i^\circ - \Sigma_{k=1}^{k=m}\Theta_{ik}^M\hat{P}_k^M}{1 - \Sigma_{k=1}^{k=m}\Theta_{ik}^M} = \hat{P}_i^\circ + \frac{\Sigma_{k=1}^{k=m}\Theta_{ik}^M(\hat{P}_i^\circ - \hat{P}_k^M)}{1 - \Sigma_{k=1}^{k=m}\Theta_{ik}^M}$$

$$(24)\quad \hat{Q}_v^i = \frac{\Sigma_{j=1}^{j=d}\Theta_{ij}^D\hat{D}_j^i}{\Sigma_{j=1}^{j=d}\Theta_{ij}^D}$$

where $\Theta_{ik}^M = (P_k^M M_k^i)/(P_i^\circ F^i)$ = the competitive share of kth imported input in ith output and $\Theta_{ij}^D = (W_j D_j^i)/(P_i^\circ F^i)$ = the competitive share of jth domestic primary input in the ith output.

If protection is conferred only on industry 1 (i.e., $\hat{P}_1^\circ \geq \hat{P}_k^M \geq \hat{P}_i^\circ$, $k = 1,\ldots,m$; $i=2,\ldots,n$, with at least one strict inequality), it can be seen from (23) that this structure results in $\hat{P}_v^1 > \hat{P}_v^i$, $i = 2,\ldots,n$.

Furthermore, we have already proved that $\dot{D}^1$ is semi-positive under this tariff structure. Hence, we see from (20), (21) and (24) that

(25) $\dot{F}^1 > 0$, $\dot{V}^1 > 0$, and $\hat{Q}_v^1 > 0$.

Hence, if protection is conferred only on industry 1 by the change in the tariff structure, then there will be a semi-positive resource inflow into this industry, and its gross output, nominal value added and $Q_v^1$ will go up. ERP theory will work, in the sense defined by B-S.

## 5. Mathematical Appendix

We are concerned with the Hessian matrix $A^i (i = 1,\ldots,n)$ of order d which has by (A-1) and (A-2) the following property:

(H) $\begin{cases} \text{(i)} & A^i \text{ is indecomposable, symmetric and singular.} \\ \text{(ii)} & A^i \text{ is negative semi-definite with negative diagonal and nonnegative off-diagonal elements.} \end{cases}$

The following additional notation is needed.

D: the set $\{1,\ldots,d\}$

J: a subset of D

$\bar{J}$: a complement of J relative to D

$A_{J\bar{J}}$: a submatrix of a matrix A which consists of the ith row and the jth column of A for $i \in J$ and $j \in \bar{J}$

$x_J$: a subvector of a vector x which consists of the ith component of x, $i \in J$

$\phi$: the empty set

First, we shall prove the following theorem on a matrix with property (H).

<u>Theorem 1:</u> If a matrix A has property (H), then the following statements hold.

(i) the equation

(1) $Ax = 0$ and $e'x = 1$

has a unique positive solution x.

(ii) the inequality

(2) $Ax \leq 0$ (or $\geq 0$)

has no solution.

(iii) any principal submatrix of A (except A itself) is negative definite.

Proof: (i) Since A is singular, the equation

(3) $Ax = 0$

has a non-zero solution x. Let $J = \{j: x_j > 0\}$ and $\bar{J} = \{j: x_j \leq 0\}$. We can assume that $J \neq \phi$, since equation (3) is homogeneous. We get from (3)

(4) $A_{JJ}x_J = -A_{J\bar{J}}x_{\bar{J}} \geq 0$

(5) $A_{\bar{J}\bar{J}}x_{\bar{J}} = - A_{\bar{J}J}x_J \leq 0$

since A is indecomposable.

Suppose $\bar{J} \neq \phi$. Then we get from (5) that

(6) $(x_{\bar{J}})'A_{\bar{J}\bar{J}}x_{\bar{J}} \geq 0$

But since $A_{\bar{J}\bar{J}}$ is negative semi-definite, we see from (6) that $(x_{\bar{J}})'A_{\bar{J}\bar{J}}x_{\bar{J}} = 0$ and therefore, $A_{\bar{J}\bar{J}}x_{\bar{J}} = 0$, a contradiction. Hence equation (3) has a positive solution x.

Suppose equation (1) has two distinct positive solutions $x^\circ$ and $\bar{x}$ ($x^\circ \neq \bar{x}$). Then $\hat{x} = x^\circ - \bar{x}$ is also a solution to equation (3). But since $e'x^\circ = e'\bar{x} = 1$, $\hat{x}$ has at least one negative component and at least one positive one, a contradiction with the above discussion. Hence, equation (1) has a unique positive solution x.

(ii) By duality,[5] (i) implies that the inequality (2) has no solution.

(iii) Since equation (3) has a positive solution x, we have for any non-empty proper subset J of D that

$$(7) \qquad A_{JJ}x_J = - A_{J\bar{J}}x_{\bar{J}} \leq 0.$$

We see from (7) that any principal submatrix of A (except A itself) has a q.d.d.[6] and therefore, is negative definite.

Q.E.D.

<u>Theorem 2:</u> $\Delta$ is negative definite if and only if the rank of the primary input coefficient matrix $[D^1,\ldots,D^n]$ is n.

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5 Exactly one of the following alternatives holds. Either the equation

$$(1) \qquad xA = 0$$

has a positive solution, or the inequality

$$(2) \qquad Av \geq 0$$

has a solution. (See Gale (1960, Theorem 2.9, Corollary 2, page 49).)

6 Professor McKenzie has revised the definition of a matrix with a quasi-dominant diagonal (or q.d.d.) made in his paper (1960) as follows. Let A be an n x n matrix and $A_{JJ}$ be the principal submatrix with indices in J. A is said to have a q.d.d. if there exist $d_i > 0$, $i = 1,\ldots,n$ such that for any principal submatrix $A_{JJ}$, $\quad d_j|a_{jj}| \geq \sum_{i \in J, \neq j} d_i|a_{ij}|$ with strict inequality for some $j \in J$

Proof: (sufficiency) It is clear that $\Delta$ is negative semi-definite. Suppose $\Delta$ is singular. Then there exists a non-zero vector $x' = ((x^2)',\ldots,(x^n)')$ such that

$$(1) \qquad x'\Delta x = ((x^2)',\ldots,(x^n)')\begin{bmatrix} A^2 + A^1 & A^1 & \cdots & A^1 \\ A^1 & A^3 + A^1 & \cdots & A^1 \\ \vdots & \vdots & & \vdots \\ A^1 & A^1 & \cdots & A^n + A^1 \end{bmatrix}\begin{bmatrix} x^2 \\ x^3 \\ \vdots \\ x^n \end{bmatrix}$$

$$= \sum_{i=2}^{n} (x^i)'A^i x^i + \left(\sum_{i=2}^{n} (x^i)'\right)A^1\left(\sum_{i=2}^{n} x^i\right) = 0$$

where $x^i$ $(i=x,\ldots,n)$ is a column vector of order d. Since $A^i$ $(i=1,\ldots,n)$ is negative semi-definite, (1) implies that

$$(2) \qquad x^i A^i x^i = 0 \quad \text{for } i = 2,\ldots,n$$

and

$$(3) \qquad \left(\sum_{i=2}^{n} (x^i)'\right)A^1\left(\sum_{i=2}^{n} x^i\right) = 0$$

Furthermore, we see from Theorem 1 that (2) and (3) imply

$$(4) \qquad x^i = \alpha_i D^i \quad (i = 2,\ldots,n) \quad \text{for some real number } \alpha_i$$

$$(5) \qquad \alpha_1 D^1 = \sum_{i=2}^{n} \alpha_i D^i$$

where $(\alpha_1,\ldots,\alpha_n) \neq 0$. But (5) implies that the rank of $[D^1,\ldots,D^n]$ is less than n, a contradiction.

(necessity) Suppose that the rank of $[D^1,\ldots,D^n]$ is less than n. Then there exists a non-zero vector $(\alpha_1,\ldots,\alpha_n)$ such that

(6) $\alpha_1 D^1 + \ldots + \alpha_n D^n = 0$

Let $x^i = \alpha_i D^i$ and $x = (x^2, \ldots, x^n)$. Then we see

(7) $x'\Delta x = (\sum_{i=2}^{n} \alpha_i (D^i)')A^1(\sum_{i=2}^{n} \alpha_i D^i) = (\alpha_1)^2 (D^1)' A^1 D^1 = 0$

(7) implies $\Delta$ is singular, a contradiction.

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